Section 4.4 Related Rates (Minimum Homework: 1, 3, 5, 7)

Suppose we have two quantities, which are connected to each other and both changing with time. A **related rates problem** is a problem in which we know the rate of change of one of the quantities and want to find the rate of change of the other quantity.

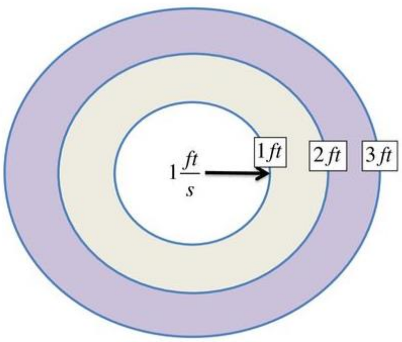
Any related rates problem can be solved as follows: (Drawing a picture can often be useful)

1. Identify the information that is given.
2. Identify what needs to be solved for.
3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)
4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.
5. Solve for the unknown rate of change.
6. Substitute all known values to get the final answer.

Let us attempt to solve problem 2 from the homework.

2) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius$r$ of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 3 feet, at what rate is the total area of the disturbed water changing?

Here is a picture that models the problem.



1. Identify the information that is given.

Let $r = outer\ radius$

$$\frac{dr}{dt} = rate\ of\ change\ the\ radius\ is\ growing\ in\ feet\ per\ second$$

Given:

$$\frac{dr}{dt} = 1\ \ foot\ per\ second$$

$$r = 3\ ft$$

2. Identify what needs to be solved for.

$We\ are\ asked\ to\ find\ the\ rate\ at\ which\ the\ total\ area\ of\ the\ disturbed\ v$

Symbolically, we are asked to find $\frac{dA}{dt}$ where A represents the disturbed area.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

$We\ need\ the\ area\ formula\ for\ a\ circle$:

$A = \pi r^2$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$

$1\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

5. Solve for the unknown rate of change.

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

6. Substitute all known values to get the final answer.

Substitute $r = 3 \ \ and \frac{dr}{dt} = 1$

$\frac{dA}{dt} = 2\pi(3)(1)$

$\frac{dA}{dt} = 6\pi$

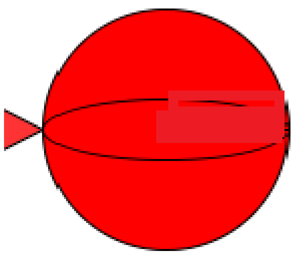
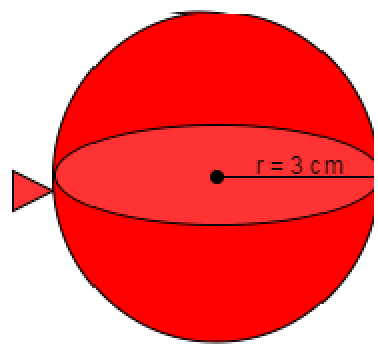
Now write an answer with units.

Answer: The disturbed area is growing at a rate of
$6\pi \ feet \ squared \ per \ second$

Let us attempt to solve problem 6 from the homework.

6) Air is being pumped into a spherical balloon at 2 cm^3/second. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 3 cm.

Here is a diagram that models the problem.

This is the baloon at some time before the radius is 3 cm 	Air is added at 2 cm^3/second $2\ cm^3 per\ second$	The radius of the balloon grows to 3 cm. 

Let us go through the steps to get this solved.

1. Identify the information that is given.

   Let $r = radius\ of\ balloon$
   $V = volume \quad (amount\ of\ air\ inside\ the\ baloon)$

Given: $r = 3\ cm$

Given: $\frac{dV}{dt} = 2\ cm^3 per\ second$

2. Identify what needs to be solved for.

We are asked to find how fast the radius is changing.

That is we are asked to find $\frac{dr}{dt}$ (hint units of our answer will be cm/sec as the length of the radius is given in centimeters, and time is given in seconds)

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

   We need the volume formula for a sphere.

   $V = \frac{4}{3}\pi r^3$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r\text{^}3\right)$

$1 * \frac{dV}{dt} = \left(3 * \frac{4}{3}\pi r^2\right)\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

5. Solve for the unknown rate of change. $we\ need\ to\ solve\ for\ \frac{dr}{dt}$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Divide both sides by $4\pi r^2$

$$\frac{\frac{dV}{dt}}{4\pi r^2} = dr/dt$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$$

6. Substitute all known values to get the final answer.

Let $\frac{dV}{dt} = 2 \quad and \quad r = 3$

$$\frac{dr}{dt} = \frac{2}{4\pi(3)^2} = \frac{2}{36\pi} = \frac{1}{18\pi}$$

The units in my answer should be centimeters per second, as the radius is measured in centimeters and the time is given in seconds.

Answer: Radius is growing at a rate of $\frac{1}{18\pi}\ centimeters\ per\ second$

1) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area of the disturbed water changing?

2) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 3 feet, at what rate is the total area of the disturbed water changing?

1. Identify the information that is given.

2. Identify what needs to be solved for.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

5. Solve for the unknown rate of change.

6. Substitute all known values to get the final answer.

$answer\text{: } radius\ growing\ 6\pi\ feet\ per\ second$

3) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 3 feet per second. When the radius is 5 feet, at what rate is the total area of the disturbed water changing?

4) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 4 feet per second. When the radius is 7 feet, at what rate is the total area of the disturbed water changing?

1. Identify the information that is given.

2. Identify what needs to be solved for.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

5. Solve for the unknown rate of change.

6. Substitute all known values to get the final answer.

$answer\text{: } radius\ growing\ 56\pi\ feet\ per\ second$

5) Air is being pumped into a spherical balloon at 10 cm^3/minute. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 6 cm.

6) Air is being pumped into a spherical balloon at 2 cm^3/second. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 3 cm.

1. Identify the information that is given.

2. Identify what needs to be solved for.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

5. Solve for the unknown rate of change.

6. Substitute all known values to get the final answer.

$answer: \ radius \ is \ growing \ by \ \frac{1}{18\pi} centimeters \ per \ secon$

7) Air is being pumped into a spherical balloon at 3 cm^3/minute. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 8 cm.

8) Air is being pumped into a spherical balloon at 9 cm^3/second. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 12 cm.

1. Identify the information that is given.

2. Identify what needs to be solved for.

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

5. Solve for the unknown rate of change.

6. Substitute all known values to get the final answer.

$$answer: \ radius \ is \ growing \ by \ \frac{1}{64\pi} centimeters \ per \ second$$